Calculating NBA Contracts with Signing Bonuses

Larry Coon
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This paper discusses how NBA contracts with signing bonuses are calculated. Correctly calculating salaries in these contracts can be difficult, since there are concurrent limits on both the first year salary (including bonus) and the total size of the bonus in relation to the total contract. Calculations need to be performed in a specific sequence in order to obtain the correct result. This paper explains that sequence.

Room (R) and First Year Salary (FYS)

There is always a limit to the salary a player can earn in the first year of his contract. The limit is the greatest of the following:

- If the team has cap room, the limit is the amount of cap room plus $100,000, not to exceed the player's maximum salary.
- If the player is signed using any exception other than the Larry Bird exception, the limit is the amount of that exception.
- If the player is signed using the Larry Bird exception, the limit is the player's maximum salary.

This limit is referred to as the Room (R) for the contract. The First Year Salary (FYS), including any amounts that are charged to the cap in the first year of the contract, cannot exceed R.

The salary in subsequent years of the contract is dependent on FYS and the limitations on raises, and not directly on R. In other words, R applies only to the first year of the contract.

Raise Factor (RF) and contracts without signing bonuses

There is a ratio between the First Year base Salary (FYS) and the Total Base Salary (TBS), based on the raise percentage (RP) and the number of years (Y). For example, if FYS is $10M and raises are 10%, then TBS in a five year contract is:

$10M + $11M + $12M + $13M + $14M = $60M

Here RF is 6.0; in other words, the TC is FYS times 6.0. This is equivalent to:

1.0*FYS + 1.1*FYS + 1.2*FYS + 1.3*FYS + 1.4*FYS = 6.0*FYS

We can also say that:

$$TBS = \sum_{i=1}^{Y} FYS + (FYS * RP * (i-1))$$

In other words, for each year from year 1 to year Y (where Y is the total number of years), the player earns an amount equal to the first year salary plus his raise. His raise is the percentage (RP) of the first year salary, times the number of years since year one (in year two it's one times the raise, in year three it's two times the raise, etc.). Add those all up, and you get the total base salary (TBS).

To calculate just the raise factor, divide the above by FYS:

$$RF = \sum_{i=1}^{Y} 1 + RP * (i-1)$$

Which is:

$$RF = \sum_{i=1}^{Y} 1 + RP * \sum_{i=1}^{Y} i - 1$$

The first term is simply equal to Y. For the second term, we can rely on the general formula:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

So we get:

$$RF = Y + RP * \frac{(Y-1) * Y}{2}$$

For example, in a five year contract with 8% raises:

$$RF = 5 + 0.08 * \frac{(5-1) * 5}{2}$$

$$RF = 5.8$$

Without signing bonuses, TBS is FYS times RF. In the above example, if FYS is \$10M, then we know that TBS is \$58M. Conversely, we know that if TBS is \$58M, then FYS is \$10M. Contracts without signing bonuses are entirely made up of base salary, so the Raise Factor is all that is needed to calculate these contracts.

Signing Bonuses and their Effect on FYS

Players are allowed to receive signing bonuses, which can be for up to 20% of the total value of the contract[1]. For example, a $100M contract may contain a signing bonus of $20M, leaving $80M in TBS. The signing bonus is paid up-front, but is charged to the cap *pro rata* across the entire contract (not including option years or years following an Early Termination Option).

The amount of the signing bonus that is applied to the team salary in any given season is proportional to the percentage of guaranteed salary in that season. In a five year contract with no options or ETOs and all seasons 100% guaranteed, 20% of the signing bonus is charged to the team salary in each year of the contract. The signing bonus is not charged to seasons following an option or ETO, so if there is one option year, then 25% of the signing bonus is charged to the team salary in each of the first four years of the contract, and none to the fifth (option) season.

When seasons are not 100% guaranteed, the calculation becomes a little more complex. Seasons can be guaranteed anywhere from 0% to 100%, except the guarantee percentage cannot increase from one season to the next. For example, a three year contract with guarantees of 100%, 80% and 70% is allowed, but guarantees of 100%, 70% and 80% is not allowed (because the guarantee percentage rises from year two to year three).

The portion of the signing bonus that is charged to the salary cap in any season is equal to that season's guarantee percentage divided by the sum of all guarantee percentages prior to any option years. We care about this value in the first season of the contract, and will call it First year Guarantee Percentage (FGP):

$$FGP = \frac{GP_1}{\sum_{i=1}^{Y-OY} GP_i}$$

For example if three seasons are guaranteed 100%, 80% and 70% respectively, then FGP is:

$$FGP = \frac{100}{100+80+70}$$

$$FGP = \frac{100}{250}$$

[1] 17.5% in contracts offered to restricted free agents after March 1, 2006.

But if the third season was an option year, it would be:

$$FGP = \frac{100}{100 + 80}$$

$$FGP = \frac{100}{180}$$

The special case is when all seasons are 0% guaranteed. When the entire contract is non-guaranteed, the entire bonus is charged to the first season.

$$FGP = 1$$

The value that is charged to the cap in the first season is important since the sum of this value and the first year base salary cannot exceed Room. We will use the term First year Signing bonus Charge (FSC) for this value, which can be expressed as:

$$FSC = SB * FGP$$

With a hard limit (R) on the first year of the contract, FYS must be reduced when any of the signing bonus is charged to the first season. This relationship can be expressed as:

$$R = FYS + FSC$$

In other words, Room is the first year base salary plus the first year signing bonus charge. Rearranging this formula, we get:

$$FYS = R - FSC$$

So when the first year signing bonus charge (FSC) increases, the first year base salary (FYS) has to decrease.

<u>The relationship between Room, Total Salary, and Signing Bonuses</u>

The Total Contract (TC) can be defined as TBS plus SB. It is the total money earned by the player from both sources – base salary and signing bonus. SB is limited to 20% of TC.

Raises are applied only to base salary. As shown above, when SB increases, FYS must decrease. This means that raises are smaller, and consequently TBS and TC are smaller as well. If TC is smaller, then SB may also need to be reduced, since SB cannot exceed 20% of TC.

We will introduce another variable, Signing Bonus Percentage (SBP), which is the percentage of TC taken by SB. In other words:

$$SBP = \frac{SB}{TC}$$

Per the rule, SBP must be between 0.0 and 0.2 (0% and 20%).

Calculating the Signing Bonus

The Signing Bonus (SB) is limited to 20% of the Total Contract (TC) *which includes the Signing Bonus itself*. To avoid a circular calculation, SB should not be determined from TC, it should be determined from Room (R), Signing Bonus Percent (SBP), the Raise Factor (RF), and the proportion of the first year guaranteed percentage to the total guaranteed percentage (FGP). As illustrated above, Room (R) is equal to the First year Base Salary (FBS) plus the First year Signing bonus Charge (FSC):

$$R = FBS + FSC$$

Since FBS is TBS divided by RF, and FSC is FGP times SB:

$$R = \frac{TBS}{RF} + FBP * SB$$

Since TBS is the Total Contract (TC) minus SB:

$$R = \frac{TC - SB}{RF} + FGP * SB$$

Since TC is SB divided by SBP:

$$R = \frac{\frac{SB}{SBP} - SB}{RF} + FGP * SB$$

Multiply everything by RF to remove the divisor:

$$R * RF = \frac{SB}{SBP} - SB + FGP * SB * RF$$

SB can be factored out:

$$R * RF = SB * (\frac{1}{SBP} - 1 + FGP * RF)$$

Isolate SB. We now have an equation to derive the total Signing Bonus from the Room, Raise Factor, Signing Bonus Percentage, and First year Guarantee Proportion:

$$SB = \frac{R * RF}{\frac{1}{SBP} - 1 + FGP * RF}$$

Example: in a five year contract with 8% raises, RF is 5.8. If there are no option years, the first season is 100% guaranteed and all subsequent seasons are 50% guaranteed, FGP is $\frac{100}{300}$ or $\frac{1}{3}$. If Room is $5 million and the Signing Bonus is 20% of the total contract, we have:

$$SB = \frac{\$5{,}000{,}000 * 5.8}{\frac{1}{0.2} - 1 + \frac{1}{3} * 5.8}$$

$$SB = \frac{\$29{,}000{,}000}{4 + \frac{5.8}{3}}$$

$$SB = \$4{,}887{,}640.45$$

Calculating the remainder of the contract

Continuing the above example of a five year contract with 8% raises, where there are no option years, the first season is 100% guaranteed, subsequent seasons are 50% guaranteed, Room is $5 million, and there is a 20% signing bonus, we can calculate the total contract (TC) from the signing bonus:

$$TC = \frac{SB}{SBP}$$

$$TC = \frac{\$4{,}887{,}640.45}{0.2}$$

$$TC = \$24{,}438{,}202.25$$

The First year Signing bonus Charge (FSC) can be calculated from the Signing Bonus and the guarantee percentages:

$$FSC = SB * \frac{GP_1}{\sum_{i=1}^{Y-OY} GP_i}$$

$$FSC = \$4{,}887{,}640.45 * \frac{100}{300}$$

$$FSC = \$1{,}629{,}213.48$$

Knowing the Room (R) and the First year Signing bonus Charge (FSC), we can determine the First Year base Salary (FYS):

$$FYS = R - FSC$$

$$FYS = \$5{,}000{,}000 - \$1{,}629{,}213.48$$

$$FYS = \$3{,}370{,}786.52$$

The Total Base Salary (TBS) is First Year Salary (FYS) times the Raise Factor (RF):

$$TBS = FYS * RF$$

$$TBS = \$3{,}370{,}786.52 * 5.8$$

$$TBS = \$19{,}550{,}561.80$$

Since we know the signing bonus should be exactly 20% of the total contract, if we add the Total Base Salary (TBS) to the Signing Bonus (SB), we should get the same Total Contract amount (TC) we got above:

$$TC = TBS + SB$$

$$TC = \$19{,}550{,}561.80 + \$4{,}887{,}640.45$$

$$TC = \$24{,}438{,}202.25$$

Since we got the same value from two different calculations, we have verified that the signing bonus is 20% of the total contract.

Another Example

Let's work through one more example, start to finish.

Room: $5,854,000 (the 2009-10 MLE amount)

Seasons: 5, with a Player Option prior to the fifth season

Raises: 7.5% each season

Guarantee percentages: 100%, 100%, 80%, 50%, 50% (respectively)

Signing bonus: 16% of the total contract

From this we want to determine the base salary and signing bonus charges in each season, along with the total value of the contract. Let's start with our useful intermediate calculation, the Raise Factor (RF):

$$RF = Y + RP * \frac{(Y-1)*Y}{2}$$

$$RF = 5 + 0.075 * \frac{(5-1)*5}{2}$$

$$RF = 5.75$$

The First year Guarantee Proportion (FGP) is the ratio of the first year guarantee percentage to the sum of all guarantee percentages prior to any option years:

$$FGP = \frac{GP_1}{\sum_{i=1}^{Y-OY} GP_i}$$

$$FGP = \frac{100}{330}$$

We now have enough information to calculate the Signing Bonus (SB):

$$SB = \frac{R * RF}{\frac{1}{SBP} - 1 + FGP * RF}$$

$$SB = \frac{\$5{,}854{,}000 * 5.75}{\frac{1}{0.16} - 1 + \frac{100}{330} * 5.75}$$

$$SB = \frac{\$33{,}660{,}500}{5.25 + \frac{575}{330}}$$

$$SB = \frac{\$21{,}952{,}500}{6.992424242}$$

$$SB = \$4{,}813{,}852.66$$

The Total Contract (TC) is the Signing Bonus (SB) divided by the Signing Bonus Percentage (SBP):

$$TC = \frac{SB}{SBP}$$

$$TC = \frac{\$4{,}813{,}852.66}{0.16}$$

$$TC = \$30{,}086{,}579.09$$

The First year Signing bonus Charge (FSC) is the First year Guarantee Percentage (FGP) times the Signing Bonus (SB):

$$FSC = FGP * SB$$

$$FSC = \frac{100}{330} * \$4{,}813{,}852.66$$

$$FSC = \$1{,}458{,}743.23$$

The First Year base Salary (FYS) is Room (R) minus the First year Signing bonus Charge (FSC):

$$FYS = R - FSC$$

$$FYS = \$5{,}854{,}000 - \$1{,}458{,}743.23$$

$$FYS = \$4{,}395{,}256.77$$

Here's what we have of the contract so far:

Year	Base Salary	Bonus	Total
1	$4,395,256.77	$1,458,743.23	$5,854,000.00
2			
3			
4			
5			
Total		$4,813,852.66	$30,086,579.09

The Signing bonus Charge for any year *n* (SC_n) is the Signing Bonus (SB) times the ratio of that year's Guarantee Percentage (GP_n) to the sum of all Guarantee Percentages preceding any option years:

$$SC_n = SB * \frac{GP_n}{\sum_{i=1}^{Y-OY} GP_i}$$

For example, in Year 2 it is the Signing Bonus times the ratio of the Year 2 Guarantee Percentage to the sum of all Guarantee Percentages preceding any option years:

$$SC_2 = SB * \frac{GP_2}{\sum_{i=1}^{\text{Y-OY}} GP_i}$$

$$SC_2 = \$4{,}813{,}852.66 * \frac{100}{330}$$

$$SC_2 = \$1{,}458{,}743.23$$

We can use the same method to find the signing bonus charges in years 3-4, and add those values to our chart (since year 5 is an option year, none of the bonus is applied to that year):

Year	Base Salary	Bonus	Total
1	$4,395,256.77	$1,458,743.23	$5,854,000.00
2		$1,458,743.23	
3		$1,166,994.58	
4		$729,371.62	
5		$0	
Total		$4,813,852.66	$30,086,579.09

Since raises in each season are 7.5% of FYS, we can determine the base salary in years 2-5 and calculate the total:

Year	Base Salary	Bonus	Total
1	$4,395,256.77	$1,458,743.23	$5,854,000.00
2	$4,724,901.03	$1,458,743.23	
3	$5,054,545.29	$1,166,994.58	
4	$5,384,189.54	$729,371.62	
5	$5,713,833.80	$0	
Total	$25,272,726.43	$4,813,852.66	$30,086,579.09

As a check, we can verify that the Total Base Salary (TBS) is the First Year base Salary (FYS) times the Raise Factor (RF):

$$TBS = FYS * RF$$

$$\$25,272,726.43 = \$4,395,256.77 * 5.75$$

$$\$25,272,726.43 = \$25,272,726.43$$

We can complete the contract by summing across the columns:

Year	Base Salary	Bonus	Total
1	$4,395,256.77	$1,458,743.23	$5,854,000.00
2	$4,724,901.03	$1,458,743.23	$6,183,644.26
3	$5,054,545.29	$1,166,994.58	$6,221,539.87
4	$5,384,189.54	$729,371.62	$6,113,561.16
5	$5,713,833.80	$0	$5,713,833.80
Total	$25,272,726.43	$4,813,852.66	$30,086,579.09

One final check – remember that we determined the Total Contract (TC = $30,086,579.09) before we filled in the row and column totals. These totals should both sum to $30,086,579.09:

TC = TBS + SB

$30,086,579.09 = $25,272,726.43 + $4,813,852.66

$30,086,579.09 = $30,086,579.09

$30,086,579.09 = $5,854,000.00 + $6,183,644.26 + $6,221,539.87 + $6,113,561.16 + $5,713,833.80

$30,086,579.09 = $30,086,579.09

Contract Calculator Spreadsheet

A spreadsheet is available at http://members.cox.net/lmcoon/Contracts.xls which performs all calculations described in this paper, based on user input for Room (size of exception or maximum salary), total years, number of Option or ETO years, raise percentage, signing bonus percentage, and the percent of guaranteed salary in each season.